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Gravitational Holography and Trapped Surfaces

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We have previously discussed the characteristics of the gravitational waves (GW) and have, theoretically, shown that, like the corresponding electromagnetic (EM) waves, they also demonstrate, under certain conditions, holographic properties. In this work we have expanded this discussion and show that the assumed gravitational holographic images may, theoretically, be related to another property of GW's which is their possible relation to singular (or nonsingular) trapped surfaces. We also show that this possibility may be, theoretically, related even to weak GW's.

KEY WORDS: gravitational waves; holography; trapped surfaces.

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1. INTRODUCTION

It is accepted in the literature that no gravitational wave (GW) (Misner *et al.*, 1973; Thorne, 1980a,b) passes a spacetime region without leaving its fingerprints in this region (Brill and Lindquist, 1963; Eppley, 1977; Tipler, 1980; Urtsever, 1988a,b; Beig and Murchadha, 1991; Abrahams and Evans, 1992; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981). As emphasized in Urtsever (1988a,b) each GW is characterized by its own intrinsic spacetime the geometry of which is imprinted upon the passed region in the sense that its geometry assume the same form as that of the GW. The imprinted geometry may be either stable for a long time if the relevant GW is strong or transient if it is weak (Eppley, 1977; Beig and Murchadha, 1991; Abrahams and Evans, 1992; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981). This geometry is, theoretically, traced and located in the related trapped surface (Brill and Lindquist, 1963; Eppley, 1977; Abrahams and Evans, 1992; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981). Among the different kinds of these surfaces one may find either the singular trapped ones (Eppley, 1977; Tipler, 1980; Urtsever, 1988a,b; Abrahams and Evans, 1992) or the nonsingular ones (Beig and Murchadha, 1991) which are related to the regular and asymptotically

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flat initial data (Misner *et al.*, 1973; Eppley, 1977; Brill, 1959, 1964; Brill and Hartle, 1964; Arnowitt *et al.*, 1962; Nakamura, 1984) in vacuum. Both kinds of these surfaces are related to strong GW's where, as mentioned, weak ones have only a transient influence upon spacetime. One may note the extensive numerical work regarding, especially, the collapse of source-free GW's in vacuum (Eppley, 1977; Abrahams and Evans, 1992; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981; Brill, 1959, 1964; Brill and Hartle, 1964; Anninos, 1997) to black holes with the accompanying apparent horizons (Eppley, 1977; Abrahams and Evans, 1992; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981; Hawking and Ellis, 1973).

In Bar (2005) we have compared the EM theory with the linearized version of general relativity and have arrived at a possible theory of gravitational wave holography in which the subject, the reference (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) and the reconstructing (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) waves are all GW's. We note that a more thorough comparison between the EM theory and the linearized version of general relativity has led Kuchar in Kuchar (1971) to the concept of extrinsic time which is canonically conjugate to spatial coordinates and not to the energy as is the usual intrinsic time. The basis of the comparison in Bar (2005) which leads to the assumed holographic properties for GW's is the phase difference and the interference which are found to exist for both EM (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) and gravitational (Bar, 2005) plane waves. This may be related to the conclusions in Tipler (1980); Urtsever (1988a,b) that the collision (corresponds to interference) between two plane waves results with a strengthening (corresponds to constructive interference) of them with the consequence of forming a singularity in the related region which is, generally, surrounded by a surface. Thus, the gravitational holographic image which is characterized by:

(1) The same spacetime geometry as that of the generating GW (Bar, 2005). (2) Changing the spacetime curvature in the region containing it relative to neighbouring regions. And (3) is formed from the constructive interference of two plane waves corresponds to the trapped surface (Eppley, 1977; Hawking and Ellis, 1973) which, likewise, (1) has the same spacetime geometry as that of the forming GW (Eppley, 1977; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981). (2) Denotes a change in spacetime curvature in the region which contains it (Brill and Lindquist, 1963; Eppley, 1977). And (3) results also from the strengthening collision of two plane waves (Tipler, 1980; Urtsever, 1988a,b). This correspondence may lead one to adopt for gravitational holography the same conclusions and methods applied for trapped surfaces, such as, for example, calculating their intrinsic geometry.

We note that although we refer here, as the basis for any linearized discussion of general relativity, to weak GW's, which entails only a small transient change in spacetime curvature (Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999;

Miyama, 1981) this small change may, however, persists if the GW which gives rise to it stays in the related region. This is the same as the optics holograpic images which are seen only when the reconstructing (reference) wave is sent through the hologram. That is, there exists a strong correspondence between the (visualized) material of the optics holographic image which is made from the light of the EM wave and the material of the gravitational holographic image (trapped surface) which is related to the mass of the GW. Note that this mass were shown to be real and positive even for a source-free GW in vacuum (Brill and Lindquist, 1963; Eppley, 1977) and without it no change in curvature results.

The mentioned trapped surfaces may, in principle, be visualized by drawing embedding diagrams (Misner *et al.*, 1973; Brill and Lindquist, 1963; Eppley, 1977; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981) of their geometry. As noted in Eppley (1977) it is a difficult task to embed the whole trapped surface but one can manage to embed the equatorial plane especially if it has rotational symmetry such as the initial data of Brill (Eppley, 1977; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981; Brill, 1959, 1964; Brill and Hartle, 1964) or the Kuchar's cylindrical ones (Kuchar, 1971). Some embedding methods may be found, for example, in Brill (1959, 1964); Brill and Hartle (1964) or Eppley (1977). We show here that the gravitational holographic image, which (1) result from the constructive interference of plane GW's (Bar, 2005) and (2) are restricted to small regions (Bar, 2005), may be regarded as a kind of trapped surface (Brill and Lindquist, 1963; Eppley, 1977). For this purpose we have calculated, using the methods in Eppley (1977), the geometry of the equatorial plane of the related holographic image.

In Section 2 we calculate the relevant expressions, such as the metrics, polarization and locations of test particles (TP), for the linearized plane GW in the transverse traceless (TT) gauge (Misner *et al.*, 1973). The mathematical process (Misner *et al.*, 1973), leading to the calculated locations of TP's, is introduced, for completness, in Appendix A. In Section 3 we calculate the relevant intensity, exposure and transmittance for the GW's which constitute the gravitational holographic process introduced in Bar (2005). In Section 4 we use the methods in Eppley (1977) for calculating and introducing the appropriate embedded trapped surfaces (Eppley, 1977; Brill and Lindquist, 1963) related to the discussed gravitational holography. The corresponding embedding, resulting under certain conditions from EM waves, is calculated in Appendix B. As mentioned, the basis for the assumed gravitational holography was the comparison in Bar (2005) between EM theory and the linearized version of general relativity. This comparison, beginning from the initial separate waves and their interference, continuing with their related intensity, exposure and transmittance and ending with the corresponding trapped surfaces is demonstrated in Table I. We summarize our results with a Concluding Remarks Section.

2. THE LINEARIZED PLANE GRAVITATIONAL WAVE

A gravitational wave is known to be characterized (Misner *et al.*, 1973; Thorne, 1980b) by an oscillating curvature tensor which causes the immediate neighbourhood of the space-time region through which it pass to correspondingly oscillate (Misner *et al.*, 1973). An example of such an oscillation is shown in Fig. 1 where a circular array of TP's changes its form to an elliptic one. If the relevant region includes the location of any two neighbouring TP's, which move along geodesic lines, then the interval between these lines also oscillates. We discuss two representative TP's, denoted $\mathcal{A}$ and $\mathcal{B}$, each traversing its own geodesic denoted also by the same $\mathcal{A}$ and $\mathcal{B}$. The separation between $\mathcal{A}$ and $\mathcal{B}$ is denoted by the vector **n**. The calculations here, such as the proper locations of $\mathcal{B}$ are done with respect to the proper reference frame of $\mathcal{A}$. That is, the spatial origin $x^j = 0$ is located on the world line of $\mathcal{A}$ and the coordinate time x^0 is identical to its proper time, e.g., $x^0 = \tau_A$. The system is also assumed to be nonrotating and so it may be considered to be a local Lorentz frame (Misner *et al.*, 1973; Bergmann, 1976) along the whole world line of $\mathcal{A}$ and not just at one event of it. As mentioned, we discuss here weak GW's and the corresponding linearized theory of gravitation.

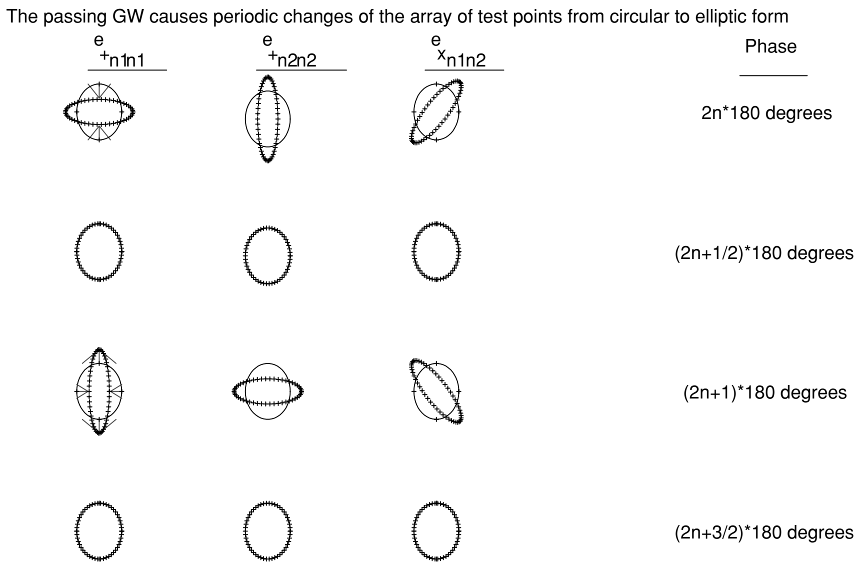

Fig. 1. A schematic representation of the influence of a passing plane GW upon a circular array of test particles which periodically changes its form to elliptic array. The first column at the left shows the influence of the unit polarization tensor $\mathbf{e}_{+\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}$, the middle column shows that of the unit polarization tensor $\mathbf{e}_{+\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}$ and the right column represents $\mathbf{e}_{\times\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}$. Note that $\mathbf{e}_{+\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1} = -\mathbf{e}_{+\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}$.

Thus, one may write the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h_{\mu\nu})^2, \tag{1}$$

where $\eta_{\mu\nu}$ is the Lorentz metric tensor (Misner *et al.*, 1973; Bergmann, 1976) and $h_{\mu\nu}$ is a small metric perturbation which, as emphasized in Misner *et al.* (1973), denotes also the passing GW which is itself a traveling perturbation in spacetime (Misner *et al.*, 1973; Thorne, 1980b). The metric, therefore, is (Misner *et al.*, 1973)

$$ds^2 = -dx^{0^2} + \delta_{jk}\, dx^j dx^k + O(|x^j|^2)\, dx^\alpha dx^\beta. \tag{2}$$

We use the transverse-traceless (TT) gauge (Misner *et al.*, 1973; Thorne, 1980b) in which the tensor $h_{\mu\nu}$ is considerably simplified and reduces to a minimum number of components (Misner *et al.*, 1973). In this gauge the components of the metric tensor satisfy (1) $h^{TT}_{\mu 0} = 0$, that is, any component of the metric tensor, except the spatial ones, vanishes, (2) $h^{TT}_{kj,j} = 0$, so that these components are divergence-free and (3) are also trace-free, e.g., $h^{TT}_{kk} = 0$. Thus, since, as mentioned, the gravitational wave is the same as h^{TT}_{jk} it, naturally, has the same properties.

The relevant calculations for the changed location (due to the passing GW) of $\mathcal{B}$ relative to $\mathcal{A}$ are introduced in Misner *et al.* (1973) and repeated, for completness, in Appendix A. The expression for this location is

$$x^j_{\mathcal{B}}(\tau) = x^k_{\mathcal{B}(0)} \left(\delta_{jk} + \frac{1}{2} h^{TT}_{jk}\right)_{at\,\mathcal{A}} \tag{3}$$

In the following we refer to a plane monochromatic gravitational wave advancing in the general $\hat{\mathbf{n}}$ direction where the TP's $\mathcal{A}$ and $\mathcal{B}$ and the geodesics along which they propagate lie in the plane perpendicular to $\hat{\mathbf{n}}$. Denoting the two perpendicular directions to $\hat{\mathbf{n}}$ by $\mathbf{e}_{\hat{\mathbf{n}}_1}$, $\mathbf{e}_{\hat{\mathbf{n}}_2}$ and comparing the polarization of the GW to that of the EM waves one may realize (Misner *et al.*, 1973) that to the unit polarization vectors $\mathbf{e}_{\hat{\mathbf{n}}_1}$ and $\mathbf{e}_{\hat{\mathbf{n}}_2}$ of the electromagnetic linearly polarized wave, which propagates in the $\hat{\mathbf{n}}$ direction, there correspond the following gravitational unit linear-polarization tensors

$$\begin{aligned} \mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} &= \mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1} - \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2} = -(\mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2} - \mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1}) = -\mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} \\ \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}} &= \mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2} + \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1} = (\mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1} + \mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2}) = \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}}, \end{aligned} \tag{4}$$

where $\otimes$ denotes the tensor product. In the following we denote by $\mathbf{r}$ the position vector of a point in space and its components by $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$. Thus, taking into account, as mentioned after Eq. (2), that in the TT gauge the following constraints hold $h^{TT}_{\mu 0} = 0$, $h^{TT}_{ij,j} = 0$, $h^{TT}_{kk} = 0$ one realizes, using Eq. (4), that, for the GW propagating in the $\hat{\mathbf{n}}$ direction, the only nonzero components which remain are

(Misner *et al.*, 1973)

$$
\begin{aligned}
h^{TT}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} &= \Re(A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{n}}}) \\
&= A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} \times \cos(k(\mathbf{r}_1 \cos(\alpha) + \mathbf{r}_2 \cos(\beta) + \mathbf{r}_3 \cos(\eta)) - ft) \\
&= -h^{TT}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} = -\Re(A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{n}}}) \\
&= -A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} \times \cos(k(\mathbf{r}_1 \cos(\alpha) + \mathbf{r}_2 \cos(\beta) + \mathbf{r}_3 \cos(\eta)) - ft)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
h^{TT}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}} &= \Re(A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{n}}}) \\
&= A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}} \times \cos(k(\mathbf{r}_1 \cos(\alpha) + \mathbf{r}_2 \cos(\beta) + \mathbf{r}_3 \cos(\eta)) - ft) \\
&= h^{TT}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}} = \Re(A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{n}}}) \\
&= A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}} \times \cos(k(\mathbf{r}_1 \cos(\alpha) + \mathbf{r}_2 \cos(\beta) + \mathbf{r}_3 \cos(\eta)) - ft)
\end{aligned} \tag{6}
$$

where $\Re$ denotes the real parts of the expressions which follow. The amplitudes A_+ and $A_\times$ are related, respectively, to the two independent modes of polarization $\mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}$ $(= -\mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}})$ and $\mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}$ $(= \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}})$, k is $\frac{2\pi}{\lambda}$, f is the time frequency, and $\cos(\alpha)$, $\cos(\beta)$, $\cos(\eta)$ are the direction cosines of $\hat{\mathbf{n}}$. Thus, one may write the perturbation h^{TT}_{jk}, resulting from the passing GW, as

$$
\begin{aligned}
h^{TT}_{jk} &= h^{TT}_{+_{jk}} + h^{TT}_{\times_{jk}} = \Re((A_+ \mathbf{e}_{+_{jk}} + A_\times \mathbf{e}_{\times_{jk}}) e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{n}}}) \\
&= (A_+ \mathbf{e}_{+_{jk}} + A_\times \mathbf{e}_{\times_{jk}}) \cos(k(\mathbf{r}_1 \cos(\alpha) \\
&\quad + \mathbf{r}_2 \cos(\beta) + \mathbf{r}_3 \cos(\eta)) - ft)
\end{aligned} \tag{7}
$$

The effect of the perturbation upon the interval between two TP's may best be understood by considering a large number of TP's $\mathcal{B}$ which form a closed ring around the TP $\mathcal{A}$ in the center. The effect of the passing wave with either $\mathbf{e}_+$ or $\mathbf{e}_\times$ polarization upon the ring is shown in Fig. 1. From this figure one may realize that circular array of the TP's $\mathcal{B}$ is periodically changed by the passing plane GW to elliptic one. These periodic changes depend upon the phase of the GW as shown in the figure. Since we discuss here several different GW's such as the subject and the reference ones we denote these waves by the appropriate suffixes S (for subject) and R (for reference). Substituting from Eq. (7) into Eq. (A.11) of Appendix A, which gives the change in the spatial interval between the TP's $\mathcal{A}$ and $\mathcal{B}$ due to the passing subject GW, one obtains for $j = \mathbf{n}_1$

$$
\begin{aligned}
x^{\hat{\mathbf{n}}_1}_{\mathcal{B}_S} = \Big\{ & x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + \frac{1}{2}\Big(A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}} x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S} \Big) \\
& \times \cos(k(x \cos(\alpha) + y \cos(\beta) + z \cos(\eta)\big) - ft)\Big\}_{at\,\mathcal{A}}
\end{aligned} \tag{8}
$$

And for $j = \mathbf{n}_2$ one obtains

$$x^{\hat{\mathbf{n}}_2}_{\mathcal{B}_S} = \Big\{ x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S} + \frac{1}{2}\left(A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}} x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + A_+ \mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S} \right) \times \cos(k(x\cos(\alpha) + y\cos(\beta) + z\cos(\eta)) - ft) \Big\}_{at\mathcal{A}} \quad (9)$$

Denoting the cosine expression $\cos(k(x\cos(\alpha) + y\cos(\beta) + z\cos(\eta)) - ft)$ by D one may realize from the last two equations that the location of the TP's $\mathcal{B}$ has been changed, due to the passage of the GW, from the initial value of $(x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S})$ to the final one of

$$x^{\hat{\mathbf{n}}}_{\mathcal{B}_S} = x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S}\left(1 + \frac{D}{2}\left(A_+\mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}} + A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}}\right)\right) + x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S}\left(1 + \frac{D}{2}\left(A_+\mathbf{e}_{+_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}} + A_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}\right)\right) = \left(x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S}\right) \times \left(1 + \frac{DA_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}{2}\right) + \left(x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} - x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S}\right)\frac{DA_+\mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}{2}, \quad (10)$$

where the last result was obtained from the first of Eq. (4). One may see from Eq. (10) that the change in the location of $\mathcal{B}$ due to the subject GW amounts to

$$\Delta x^{\hat{\mathbf{n}}}_{\mathcal{B}_S} = \left(x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} + x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S}\right)\frac{DA_\times \mathbf{e}_{\times_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}{2} + \left(x^{\hat{\mathbf{n}}_1}_{\mathcal{B}(0)_S} - x^{\hat{\mathbf{n}}_2}_{\mathcal{B}(0)_S}\right)\frac{DA_+\mathbf{e}_{+_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}{2} \quad (11)$$

As shown (Bar, 2005) this change is added to a similar change due to a second GW which constructively interfere with the former GW. The resulting sum is imprinted in spacetime in the sense that a circular array of TP's is changed to an elliptic one. The theory of this interference and the resulting gravitational holographic image has been detaily described in Bar (2005) and we introduce in the following section the results obtained there.

3. THE GRAVITATIONAL INTENSITY, EXPOSURE AND TRANSMITTANCE

The reference wave denoted by the suffix R may be given, as done in Bar (2005), by an expression similar to Eq. (7) and the change in the location of the TP $\mathcal{B}$ due to its passage may, likewise, be written in a form similar to Eqs. (8)–(11). Thus, using Eq. (7) and the expression (Bar, 2005) for the intensity of GW, one may write the intensity of the total wave resulting from the interference of the subject and reference waves as

$$I_{(S+R)_{jk}} = \left(h^{TT}_{+_{S_{jk}}} + h^{TT}_{\times_{S_{jk}}}\right)\left(h^{TT}_{+_{S_{jk}}} + h^{TT}_{\times_{S_{jk}}}\right)^* + \left(h^{TT}_{+_{R_{jk}}} + h^{TT}_{\times_{R_{jk}}}\right)$$

$$\times\left(h^{TT}_{+_{R_{jk}}} + h^{TT}_{\times_{R_{jk}}}\right)^* + \left\langle\left(h^{TT}_{+_{S_{jk}}} + h^{TT}_{\times_{S_{jk}}}\right)\left(h^{TT}_{+_{R_{jk}}} + h^{TT}_{\times_{R_{jk}}}\right)^* + \left(h^{TT}_{+_{S_{jk}}} + h^{TT}_{\times_{S_{jk}}}\right)^*\left(h^{TT}_{+_{R_{jk}}} + h^{TT}_{\times_{R_{jk}}}\right)\right\rangle, \quad (12)$$

where the asteric denotes the conjugate part of the relevant complex expressions. In the following we append to A_+, $A_\times$, $\mathbf{e}_+$, $\mathbf{e}_\times$, $\mathbf{r}$, $\mathbf{n}$ the suffixes S and R to differentiate between the subject and reference GW's. Thus, we use (Misner *et al.*, 1973; Bar, 2005) for the subject and reference GW's the following expressions; $h^{TT}_{+_{S_{jk}}} = A_{+_S}\mathbf{e}_{+_{S_{jk}}} e^{-if_S t} e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$, $h^{TT}_{\times_{S_{jk}}} = A_{\times_S}\mathbf{e}_{\times_{S_{jk}}} e^{-if_S t} e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$, $h^{TT}_{+_{R_{jk}}} = A_{+_R}\mathbf{e}_{+_{R_{jk}}} e^{-if_R t} e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$, $h^{TT}_{\times_{R_{jk}}} = A_{\times_R}\mathbf{e}_{\times_{R_{jk}}} e^{-if_R t} e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$. Assuming, as generally done in optics holography, the same frequency $f_S = f_R = f$ for the subject and reference GW's one obtains for the average factors in the third average term in Eq. (12) $\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} e^{i(f_R-f_S)t}dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} dt = 1$. Thus, one may write Eq. (12) as

$$\begin{aligned} I_{(S+R)_{jk}} &= (A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}})(A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}})^* \\ &+ (A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}})(A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}})^* \\ &+ 2(A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}})(A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}}) \\ &\times\ \cos(ik(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R)), \end{aligned} \quad (13)$$

where the trigonometric identity $e^{ik(\mathbf{r}_S\mathbf{n}_S-\mathbf{r}_R\mathbf{n}_R)} + e^{-ik(\mathbf{r}_S\mathbf{n}_S-\mathbf{r}_R\mathbf{n}_R)} = 2\cos(k(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R))$ is used. One may realize from Eq. (13) that for $\cos(k(\mathbf{r}_R\mathbf{n}_R - \mathbf{r}_S\mathbf{n}_S)) = 0$ there is no interference at all between the GW's $h^{TT}_{+_{S_{jk}}}$, $h^{TT}_{\times_{S_{jk}}}$ and $h^{TT}_{+_{R_{jk}}}$, $h^{TT}_{\times_{R_{jk}}}$ and the total intensity $I_{(S+R)_{jk}}$ is the addition of the separate intensities $I_{S_{jk}}$ and $I_{R_{jk}}$. That is, for $(k(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R)) = \frac{N\pi}{2}$ where N are the uneven numbers $N = 1, 3, 5\ldots$ one obtains

$$\begin{aligned} I_{(S+R)_{jk(\cos(k(\mathbf{r}_S\mathbf{n}_S-\mathbf{r}_R\mathbf{n}_R))=0)}} &= (A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}})(A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}})^* \\ &+ (A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}})(A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}})^* \end{aligned} \quad (14)$$

For $\cos(k(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R)) \neq 0$ the interference between the GW's $h^{TT}_{+_{S_{jk}}}$, $h^{TT}_{\times_{S_{jk}}}$ and $h^{TT}_{+_{R_{jk}}}$, $h^{TT}_{\times_{R_{jk}}}$ does not vanish and the intensity $I_{(S+R)_{jk}}$ depends upon the value of $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R))$. Thus, for $\cos(k(\mathbf{r}_R\mathbf{n}_R - \mathbf{r}_S\mathbf{n}_S)) = \pm 1$ one have

$$\begin{aligned} I_{(S+R)_{jk(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S-\mathbf{r}_R\hat{\mathbf{n}}_R))=\pm 1)}} &= [((A_{+_S}\mathbf{e}_{+_{S_{jk}}} + A_{\times_S}\mathbf{e}_{\times_{S_{jk}}}) \\ &\pm (A_{+_R}\mathbf{e}_{+_{R_{jk}}} + A_{\times_R}\mathbf{e}_{\times_{R_{jk}}})]^2, \end{aligned} \quad (15)$$

where the $+$ sign between the two terms at the right corresponds to $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ and the $-$ sign to $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$. Equation (15) may be used in conjunction with Fig. 1 to differentiate between constructive and destructive interference. We first take into account that for constructive interference the GW's $h^{TT}_{+_{S_{jk}}}$, $h^{TT}_{+_{R_{jk}}}$ and $h^{TT}_{\times_{S_{jk}}}$, $h^{TT}_{\times_{R_{jk}}}$ must have approximately the same unit polarization tensors, e.g., $\mathbf{e}_{+_{S_{jk}}} \approx \mathbf{e}_{+_{R_{jk}}}$, $\mathbf{e}_{\times_{S_{jk}}} \approx \mathbf{e}_{\times_{R_{jk}}}$ and for destructive interference these GW's must have approximately opposite unit polarization tensors, e.g., $\mathbf{e}_{+_{S_{jk}}} \approx -\mathbf{e}_{+_{R_{jk}}}$, $\mathbf{e}_{\times_{S_{jk}}} \approx -\mathbf{e}_{\times_{R_{jk}}}$.

Thus, one may realize that the first case of approximate similar polarization for the GW's $h^{TT}_{+_{S_{jk}}}$, $h^{TT}_{\times_{S_{jk}}}$ and $h^{TT}_{+_{R_{jk}}}$, $h^{TT}_{\times_{R_{jk}}}$ must corresponds to $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ and the second case of approximate opposite polarization for these GW's corresponds to $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$. This may also be realized by using the trigonometric identity $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = \cos(k\mathbf{r}_S\hat{\mathbf{n}}_S)\cos(k\mathbf{r}_R\hat{\mathbf{n}}_R) + \sin(k\mathbf{r}_S\hat{\mathbf{n}}_S)\sin(k\mathbf{r}_R\hat{\mathbf{n}}_R)$ from which one may conclude that for $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ the angles $k\mathbf{r}_S\hat{\mathbf{n}}_S$ and $k\mathbf{r}_R\hat{\mathbf{n}}_R$ respectively related to the subject and reference GW's should be approximately the same or differ by $2\pi n$, where $n = 0, \ \pm 1, \ \pm 2, \pm 3 \ldots$. The latter relations between the angles $k\mathbf{r}_S\hat{\mathbf{n}}_S$ and $k\mathbf{r}_R\hat{\mathbf{n}}_R$ means that the polarizations of the corresponding GW's are the same and, therefore, constructively interfere. In a similar manner one may realize that for $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$ the former angles should be separated from each other by $(2n+1)\pi$, where $n = 0, \pm 1, \pm 2, \pm 3 \ldots$ which means that the corresponding polarizations of these waves are opposite to each other and, therefore, destructively interfere. Thus, since as just mentioned the two cases of $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ and $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$ respectively correspond to the same (costructive interference) and opposite (destructive interference) polarizations for the GW's $h^{TT}_{+_{S_{jk}}}$, $h^{TT}_{\times_{S_{jk}}}$ and $h^{TT}_{+_{R_{jk}}}$, $h^{TT}_{\times_{R_{jk}}}$ one may rewrite Eq. (15) as

$$I_{(S+R)_{jk_{(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R))=\pm 1}}} = (\mathbf{e}_{+_{S_{jk}}}(A_{+_S} \pm A_{+_R}) + \mathbf{e}_{\times_{S_{jk}}}(A_{\times_S} \pm A_{\times_R}))^2 \qquad (16)$$

For further substantiating the former discussion we refer to Fig. 1 and, therefore, we particularize the general indices j and k and assume, for example, $j = k = \hat{\mathbf{n}}_1$. We also denote the horizontal and vertical axes in Fig. 1 by $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ respectively. Thus, for $j = k = \hat{\mathbf{n}}_1$ and $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ Eq. (16) becomes

$$\begin{aligned} I_{(S+R)_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R))=1)}} &= (\mathbf{e}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}(A_{+_S} + A_{+_R}))^2 \\ &= ((\mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1} - \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2})(A_{+_S} + A_{+_R}))^2, \qquad (17) \end{aligned}$$

where we use the first of Eqs. (4). From the last equation one may realize that the unit polarization tensors $\mathbf{e}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ and $\mathbf{e}_{+_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ are similar to each other which means that the GW's $h^{TT}_{+_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ and $h^{TT}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ act in identical manner upon the ensemble of

TP's $\mathcal{B}$ which is to turn it from a circular form to an elliptic one so that they are constructively interfering together.

In a similar manner one may discuss the case of $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$ so that for $j = k = \hat{\mathbf{n}}_1$ Eq. (16) may be rewritten as

$$\begin{aligned} I_{(S+R)_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S-\mathbf{r}_R\hat{\mathbf{n}}_R))=-1)}} &= (\mathbf{e}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}(A_{+_S} - A_{+_R}))^2 \\ &= ((\mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1} - \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2})(A_{+_S} - A_{+_R}))^2, \end{aligned} \tag{18}$$

where we again use the first of Eqs. (4). From the last equation one may realize that, as mentioned, the unit polarization tensors $\mathbf{e}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ and $\mathbf{e}_{+_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ are opposite to each other. That is, the GW's $h^{TT}_{+_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ and $h^{TT}_{+_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1}}}$ act in contradicting manner upon the ensemble of TP's $\mathcal{B}$ so that, for $A_{+_S} \approx A_{+_R}$, the resulting action is almost null which means that they are destructively interfering with each other.

When, however, the general indices jk are particularized to $j = \hat{\mathbf{n}}_1$ and $k = \hat{\mathbf{n}}_2$ then one may follow the former discussion and conclude that for $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = 1$ the polarization tensors $\mathbf{e}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$, $\mathbf{e}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ of the respective GW's $h^{TT}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$, $h^{TT}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ are similar to each other. Thus, Eq. (16) may be rewritten in this case as

$$\begin{aligned} I_{(S+R)_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S-\mathbf{r}_R\hat{\mathbf{n}}_R))=1)}} &= (\mathbf{e}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}(A_{\times_S} + A_{\times_R}))^2 \\ &= ((\mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2} + \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1})(A_{\times_S} + A_{\times_R}))^2, \end{aligned} \tag{19}$$

where we use the second of Eqs. (4). As seen from the last equation the unit polarization tensors $\mathbf{e}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ and $\mathbf{e}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ are similar to each other which means that the GW's $h^{TT}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ and $h^{TT}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ act in identical manner upon the ensemble of TP's $\mathcal{B}$ which, as seen from Fig. 1, is composed of; (1) turning it from a circular form to an elliptic one and (2) rotating it by 45 degrees so that they are constructively interfering together. The case of $\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S - \mathbf{r}_R\hat{\mathbf{n}}_R)) = -1$ may also be discussed in a similar manner so that for $j = \hat{\mathbf{n}}_1,\ k = \hat{\mathbf{n}}_2$ Eq. (16) may be rewritten as

$$\begin{aligned} I_{(S+R)_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2(\cos(k(\mathbf{r}_S\hat{\mathbf{n}}_S-\mathbf{r}_R\hat{\mathbf{n}}_R))=-1)}} &= (\mathbf{e}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}(A_{\times_S} - A_{\times_R}))^2 \\ &= ((\mathbf{e}_{\hat{\mathbf{n}}_1} \otimes \mathbf{e}_{\hat{\mathbf{n}}_2} + \mathbf{e}_{\hat{\mathbf{n}}_2} \otimes \mathbf{e}_{\hat{\mathbf{n}}_1})(A_{\times_S} - A_{\times_R}))^2, \end{aligned} \tag{20}$$

where we again use the second of Eqs. (4). As realized from the last equation the unit polarization tensors $\mathbf{e}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ and $\mathbf{e}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ are opposite to each other. That is, the GW's $h^{TT}_{\times_{S_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ and $h^{TT}_{\times_{R_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2}}}$ act in contradicting manner upon the ensemble of TP's $\mathcal{B}$ so that, for $A_{\times_S} \approx A_{\times_R}$ the resulting action is almost null which means that they are destructively interfering with each other.

As for the gravitational hologram we may understand its nature, as emphasized in Bar (2005), from the known corresponding holograms used in optics holography. The latter are prepared (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) so as to efficiently record the initial constructive interference of the subject and reference waves so that directing later the reference wave upon its surface results in reconstructing the initial subject wave (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971).

A great simplification of the recording process is obtained in optics holography, as done in Collier *et al.* (1971), when one discusses a small area holograms in which case one may assume a linear recording process (Collier *et al.*, 1971). This process is also used in Bar (2005) for the gravitational case and were theoretically shown that one may reconstruct the initial subject gravitational wave. That is, as shown in Bar (2005), the imprinted spacetime of the subject GW $h^{TT}_{S_{jk}} = h^{TT}_{+_{S_{jk}}} + h^{TT}_{\times_{S_{jk}}}$ upon the small region A (see Fig. 2) becomes effective in the sense that if a reconstructing GW, which should be identical to the original reference wave, passes through this region the effect is to cause a contraction of A along some axis and elongation along another as shown in Fig. 1. This is the meaning by which gravitational holographic images should be understood. In other words, assuming as in optics holography (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971), that the spacetime region exposed to the interfering subject and reference GW's acquires some transmittance $\mathbf{t}^{TT}_E$ which depends upon this exposure E one may suppose the following:

(1) the exposure E is proportional to the intensity I_{S+R} from Eq. (13) so that $E = kI_{S+R}\tau_e$ where τ_E is the exposure time and k a proportionality constant.

(2) the exposure E may be written as a sum $E(r) = E_0 + E_1(r)$ of a constant term E_0 and a space dependent one $E_1(r)$ where the restriction to the small region A enables one to sustain the inequality $E_1(r) < E_0$ over A.

(3) Using (2) one may write the transmittance $\mathbf{t}^{TT}_E$ over A as a Taylor series in which the coefficients of the second and higher order terms may be neglected. That is

$$\mathbf{t}^{TT}_E = \mathbf{t}^{TT}\left(E^{TT}_0\right) + E^{TT}_1 \left.\frac{d\mathbf{t}_E}{dE}\right|_{E^{TT}_0} + \frac{1}{2}\left(E^{TT}_1\right)^2 \left.\frac{d^2\mathbf{t}_E}{dE^2}\right|_{E^{TT}_0} + \cdots, \quad (21)$$

where $\frac{d^2\mathbf{t}_E}{dE^2}|_{E^{TT}_0} \approx \frac{d^3\mathbf{t}_E}{dE^3}|_{E^{TT}_0} \approx\approx ....0$. Thus, as for optics hologram (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) and as shown in Bar (2005), for reconstructing the subject wave $h^{TT}_{S_{jk}}$ one should send through A a reconstructing GW, which is identical to the reference wave $h^{TT}_{R_{jk}}$, so that using

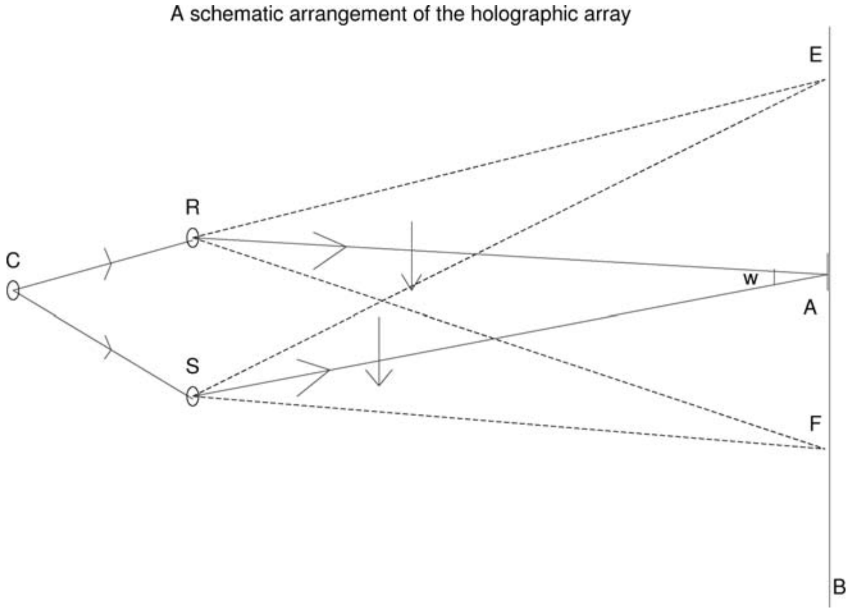

Fig. 2. The constructive interference process of the subject and reference waves over the small region A shown in the middle of the figure. Other small regions, denoted F and E are shown higher and lower than A. The lines, representing the GW's which proceed to these regions are shown in dashed forms.

(1)–(3) and Eq. (21) one obtains

$$W = h^{TT}_{R_{jk}} \mathbf{t}^{TT}_{E} = h^{TT}_{R_{jk}} E^{TT}_{1}(r) \left. \frac{d\mathbf{t}_E}{dE} \right|_{E^{TT}_0} = C_{TT} \cdot h^{TT}_{S_{jk}}, \tag{22}$$

where C_{TT} is a proportionality constant. In Table I we have outlined and followed the whole gravitational holographic process from the initial separate subject and reference GW's until the final formed trapped surface. Since, as mentioned, this process is based upon the comparison done in Bar (2005) between the optical holographic theory (Gabor, 1948, 1949, 1951; Collier *et al.*, 1971) and the linearized version of general relativity (Misner *et al.*, 1973; Thorne, 1980b) we have also described in Table I the corresponding optics holographic process. Thus, one may see in this table side by side the corresponding expressions for the two processes.

We should note here that unlike the EM field which may, experimentally, be traced and located in any region in space however small it is the gravitational field can not be located (Misner *et al.*, 1973) in such a manner. That is, as emphasized

Table I. The table compares the holographic evolutions for the electromagnetic and gravitational waves from the initial stage of following the separate subject and reference waves to the final stage of reconstructing the subject wave and forming the trapped surfaces

N	The holographic evolutions for the electromagnetic and gravitational waves	The electromagnetic wave holography	The gravitational wave holography
1	The initial subject and reference waves where $\mathbf{e}_{S_j}$, $\mathbf{e}_{S_k}$, $\mathbf{e}_{R_j}$, $\mathbf{e}_{R_k}$ are the EM polarization vectors and $\mathbf{e}_{+S_{jk}}$, $\mathbf{e}_{+S_{jk}}$, $\mathbf{e}_{\times R_{jk}}$, $\mathbf{e}_{+R_{jk}}$ are the GW polarization tensors.	$S_{jk}^{EM} = (A_{S_j}\mathbf{e}_{S_j} + A_{S_k}\mathbf{e}_{S_k})e^{-ift}e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$ $R_{jk}^{EM} = (A_{R_j}\mathbf{e}_{R_j} + A_{R_k}\mathbf{e}_{R_k})e^{-ift}e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$	$h_{S_{jk}}^{TT} = (A_{+S}\mathbf{e}_{+S_{jk}} + A_{\times S}\mathbf{e}_{\times S_{jk}})e^{-ift}e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$ $h_{R_{jk}}^{TT} = (A_{+R}\mathbf{e}_{+R_{jk}} + A_{\times R}\mathbf{e}_{\times R_{jk}})e^{-ift}e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$
2	The total intensities of the interfering subject and reference waves where $I_{(S+R)_0}^{EM}$, $I_{(S+R)_0}^{TT}$ denote the terms which do not depend upon the complex exponentials and $I_{(S+R)_1}^{EM}$, $I_{(S+R)_1}^{TT}$ denote the terms which depend upon them	$I_{S+R}^{EM} = I_{(S+R)_0}^{EM} + I_{(S+R)_1}^{EM} = \left(A_{S_j}\mathbf{e}_{S_j} + A_{S_k}\mathbf{e}_{S_k}\right)$ $\times \left(A_{S_j}\mathbf{e1}_{S_j} + A_{S_k}\mathbf{e}_{S_k}\right)^* + \left(A_{R_j}\mathbf{e}_{R_j} + A_{R_k}\mathbf{e}_{R_k}\right)$ $\times \left(A_{R_j}\mathbf{e}_{R_j} + A_{R_k}\mathbf{e}_{R_k}\right)^* + \Big[\left(A_{S_j}\mathbf{e}_{S_j} + A_{S_k}\mathbf{e}_{S_k}\right)$ $\times \left(A_{R_j}\mathbf{e}_{R_j} + A_{R_k}\mathbf{e}_{R_k}\right)\Big]e^{ik(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R)}$	$I_{S+R}^{TT} = I_{(S+R)_0}^{TT} + I_{(S+R)_1}^{TT} =$ $\left(A_{+S}\mathbf{e}_{+S_{jk}} + A_{\times S}\mathbf{e}_{\times S_{jk}}\right)\left(A_{+S}\mathbf{e}_{+S_{jk}} +\right.$ $\left.A_{\times S}\mathbf{e}_{\times S_{jk}}\right)^* + \left(A_{+R}\mathbf{e}_{+R_{jk}} +\right.$ $\left.A_{\times R}\mathbf{e}_{\times R_{jk}}\right)\left(A_{+R}\mathbf{e}_{+R_{jk}} + A_{\times R}\mathbf{e}_{\times R_{jk}}\right)^* +$ $\Big[\left(A_{+S}\mathbf{e}_{+S_{jk}} + A_{\times S}\mathbf{e}_{\times S_{jk}}\right)\left(A_{+R}\mathbf{e}_{+R_{jk}} +\right.$ $\left.A_{\times R}\mathbf{e}_{\times R_{jk}}\right)\Big]e^{ik(\mathbf{r}_S\mathbf{n}_S - \mathbf{r}_R\mathbf{n}_R)}$
3	The exposure related to the total intensities where τ_e is the exposure time	$E^{EM} = E_0^{EM} + E_1^{EM} = I_{S+R}^{EM}\tau_e =$ $I_{(S+R)_0}^{EM}\tau_e + I_{(S+R)_1}^{EM}\tau_e$	$E^{TT} = E_0^{TT} + E_1^{TT} = I_{S+R}^{TT}\tau_e =$ $I_{(S+R)_0}^{TT}\tau_e + I_{(S+R)_1}^{TT}\tau_e$
4	The optics and GW holograms expressed by the respective amplitude transmittances $\mathbf{t}_{E^{EM}}$ and $\mathbf{t}_{E^{TT}}$ as functions of the exposures E^{EM}, E^{TT} in the limit of linear recording	$\mathbf{t}_{E^{EM}} = \mathbf{t}^{EM}(E_0^{EM}) + E_1^{EM}\frac{d\mathbf{t}_E}{dE}\vert_{E_0^{EM}} +$ $\frac{1}{2}(E_1^{TT})^2\frac{d^2\mathbf{t}_E}{dE^2}\vert_{E_0^{TT}} + \cdots$	$\mathbf{t}_{E^{TT}} = \mathbf{t}^{TT}(E_0^{TT}) + E_1^{TT}\frac{d\mathbf{t}_E}{dE}\vert_{E_0^{TT}} +$ $\frac{1}{2}(E_1^{TT})^2\frac{d^2\mathbf{t}_E}{dE^2}\vert_{E_0^{TT}} + \cdots$

Table I. Continued.

N	The holographic evolutions for the electromagnetic and gravitational waves	The electromagnetic wave holography	The gravitational wave holography
5	For reconstructing the holographic EM and gravitational images the reference waves R_{jk}^{EM}, $h_{R_{jk}}^{TT}$ are sent to the respective holograms $\mathbf{t}_{EEM}$ and $\mathbf{t}_{ETT}$	$R_{jk}^{EM} = (A_{R_j}\mathbf{e}_{R_j} + A_{R_k}\mathbf{e}_{R_k})e^{-ift}e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$	$h_{R_{jk}}^{TT} = (A_{+R}\mathbf{e}_{+R_{jk}} + A_{\times R}\mathbf{e}_{\times R_{jk}})e^{-ift}e^{ik\mathbf{r}_R\hat{\mathbf{n}}_R}$
6	The reference waves R_{jk}^{EM} and $h_{R_{jk}}^{TT}$ are decomposed at the respective holograms $\mathbf{t}_{EEM}$ and $\mathbf{t}_{ETT}$ in the limit of linear recording to the holographic images S_{jk}^{EM} and $h_{S_{jk}}^{TT}$ where C_{EM} and C_{TT} are constants	$R_{jk}^{EM}\mathbf{t}_{EEM} = C_{EM}S_{jk}^{EM} = C_{EM}(A_{S_j}\mathbf{e}_{S_j} + A_{S_k}\mathbf{e}_{S_k})e^{-ift}e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$	$h_{R_{jk}}^{TT}\mathbf{t}_{ETT} = C_{TT}h_{S_{jk}}^{TT} = C_{TT}(A_{+S}\mathbf{e}_{+S_{jk}} + A_{\times S}\mathbf{e}_{\times S_{jk}})e^{-ift}e^{ik\mathbf{r}_S\hat{\mathbf{n}}_S}$
7	The geometry of the trapped surfaces resulting from the influences of the EM and gravitational waves.	$a^{EM}(\rho) = \rho\Big[\cos(kz - ft)\Big\{\Big(\sin^2(\phi)\cos(\phi) + \cos^2(\phi)\sin(\phi)\Big)\mathbf{e}_{\hat{\rho}} + \Big(\cos^3(\phi) - \sin^3(\phi)\Big)\mathbf{e}_{\hat{\phi}}\Big\}\Big]^{\frac{1}{2}}$	$a^{TT}(\rho) = \rho\Big[\cos(kz - ft)\Big\{\frac{\sin(4\phi)}{2}(A_+ - A_\times)\Big(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\Big) + \Big(A_+\cos^2(2\phi) + A_\times\sin^2(2\phi)\Big)\Big(\mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}} - \mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}}\Big)\Big\}\Big]^{\frac{1}{2}}$
	The trapped surface for the EM case results for the special condition of very small wavelength.	$a^{EM}(\rho)_\rho = \Big[\cos(kz - ft)\Big\{\Big(\sin^2(\phi)\cos(\phi) + \cos^2(\phi)\sin(\phi)\Big)\mathbf{e}_{\hat{\rho}} + \Big(\cos^3(\phi) - \sin^3(\phi)\Big)\mathbf{e}_{\hat{\phi}}\Big\}\Big]^{\frac{1}{2}}$	$a^{TT}(\rho)_\rho = \Big[\cos(kz - ft)\Big\{\frac{\sin(4\phi)}{2}(A_+ - A_\times)\Big(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\Big) + \Big(A_+\cos^2(2\phi) + A_\times\sin^2(2\phi)\Big)\Big(\mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}} - \mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}}\Big)\Big\}\Big]^{\frac{1}{2}}$
	Note that the geometries given by $a^{EM}(\rho)$, $a^{EM}(\rho)_\rho$ $b^{EM}(\rho)$, $a^{TT}(\rho)$, $a^{TT}(\rho)_\rho$, $b^{TT}(\rho)$ are for the embedding surfaces only (see text).	$b^{EM}(\rho) = \int d\rho\Big[\cos(kz - ft)\Big\{\Big(\cos^3(\phi) + \sin^3(\phi)\Big)\mathbf{e}_{\hat{\rho}} + \Big(\sin^2(\phi)\cos(\phi) - \cos^2(\phi)\sin(\phi)\Big)\mathbf{e}_{\hat{\phi}}\Big\} - (a^{EM}(\rho)_\rho)^2\Big]^{\frac{1}{2}}$	$b^{TT}(\rho) = \int d\rho\Big[\cos(kz - ft)\Big\{\frac{\sin(4\phi)}{2}(A_\times - A_+)\Big(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\Big) + \Big(A_+\cos^2(2\phi) + A_\times\sin^2(2\phi)\Big)\Big(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}}\Big)\Big\} - (a^{TT}(\rho)_\rho)^2\Big]^{\frac{1}{2}}$

in Misner *et al.* (1973) (see p. 955 in Misner *et al.* (1973)), *"the stress-energy carried by gravitational waves can not be localized inside a wavelength etc. However, one can say that a certain amount of stress-energy is contained in a given 'macroscopic' region."* That is, the gravitational wave formalism is applied for averages over several wavelengths. However, as mentioned, for very small wavelength, which is the limit discussed here, not only this formalism is valid but also the comparison between it and the EM one.

4. CALCULATION OF THE EMBEDDED SURFACE

As mentioned, the holographic image over A have a spacetime geometry which is the same as the spacetime geometry of the subject GW $h^{TT}_{S_{jk}}$. We have also mentioned that the spacetime geometry of the trapped surface is the same as that of the GW which gives rise to it (Eppley, 1977; Brill, 1959, 1964; Brill and Hartle, 1964). This similarity between the holographic images and trapped surface lead us to suppose that, theoretically, they are similar entities. Thus, one may be tempted to use the known embedding methods (Eppley, 1977; Brill, 1959, 1964; Brill and Hartle, 1964) of calculating the geometry of trapped surfaces for finding the geometry of the gravitational holographic images. Now, since the embedding of the calculated geometry is into the Euclidean space (Eppley, 1977) we have first to convert the tensor metric components $h^{TT}_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_1} = -h^{TT}_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_2}$, $h^{TT}_{\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2} = h^{TT}_{\hat{\mathbf{n}}_2\hat{\mathbf{n}}_1}$ from Eqs. (5) and (6), which were calculated in the $\hat{\mathbf{n}}$, $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$ system, into the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ Euclidean system. That is, substituting $\hat{\mathbf{n}} = \hat{\mathbf{z}}$, $\hat{\mathbf{n}}_1 = \hat{\mathbf{x}}$, $\hat{\mathbf{n}}_2 = \hat{\mathbf{y}}$, $\mathbf{r} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$ one may write the Euclidean metric components as

$$\begin{aligned}
h^{TT}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} &= \Re\left(A_+\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}e^{-ift}e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = A_+\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}\cdot\cos(kz-ft)\\
&= -h^{TT}_{\hat{\mathbf{y}}\hat{\mathbf{y}}} = -\Re\left(A_+\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}e^{-ift}e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = -A_+\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}\cdot\cos(kz-ft)\\
h^{TT}_{\hat{\mathbf{x}}\hat{\mathbf{y}}} &= \Re\left(A_\times\mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}e^{-ift}e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = A_\times\mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}\cdot\cos(kz-ft) \qquad (23)\\
&= h^{TT}_{\hat{\mathbf{y}}\hat{\mathbf{x}}} = \Re\left(A_\times\mathbf{e}_{\times_{\hat{\mathbf{y}}\hat{\mathbf{x}}}}e^{-ift}e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = A_\times\mathbf{e}_{\times_{\hat{\mathbf{y}}\hat{\mathbf{x}}}}\cdot\cos(kz-ft),
\end{aligned}$$

where $\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}$, $\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}$, $\mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}$ are the Euclidean unit linear-polarization tensors given by Eqs. (4) in which we substitute $\hat{\mathbf{n}} = \hat{\mathbf{z}}$, $\hat{\mathbf{n}}_1 = \hat{\mathbf{x}}$, $\hat{\mathbf{n}}_2 = \hat{\mathbf{y}}$. Thus, using the last equations and the discussion after Eq. (2) one may write the metric from Eq. (2) in the TT gauge as

$$\begin{aligned}
(ds^{TT})^2_{(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})} &= h^{TT}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}dx^2 + h^{TT}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}dy^2 + 2h^{TT}_{\hat{\mathbf{x}}\hat{\mathbf{y}}}dx\,dy = A_+\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}\cdot\cos(kz-ft)\,dx^2\\
&\quad + A_+\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}\cdot\cos(kz-ft)dy^2 + 2A_\times\mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}\cdot\cos(kz-ft)\,dx\,dy \qquad (24)\\
&= A_+\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}\cdot\cos(kz-ft)(dx^2-dy^2) + 2A_\times\mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}\cdot\cos(kz-ft)\,dx\,dy,
\end{aligned}$$

where the last result was obtained by using $\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}} = -\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}$ (see the first of Eqs. (4)). In order to calculate the embedded surface of the holographic image we first express the metric from Eq. (24) in the cylindrical coordinates $(\hat{\rho},\ \hat{\phi},\ \hat{\mathbf{z}})$ where $x = \rho\cos(\phi),\ y = \rho\sin(\phi),\ z = z$ so that

$$\begin{aligned}(ds^{TT})^2_{(\hat{\rho},\hat{\phi},\hat{\mathbf{z}})} &= h^{TT}_{\hat{\rho}\hat{\rho}}d\rho^2 + h^{TT}_{\hat{\phi}\hat{\phi}}d\phi^2 + h^{TT}_{\hat{\rho}\hat{\phi}}d\rho\, d\phi \\ &= A_+\mathbf{e}_{+_{\hat{\rho}\hat{\rho}}}\cdot\cos(kz - ft)(\cos(2\phi)(d\rho^2 - \rho^2 d\phi^2) \\ &\quad - 2\rho\sin(2\phi)\, d\rho\, d\phi) + A_\times\mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}} \\ &\quad \times\cos(kz - ft)(\sin(2\phi)(d\rho^2 - \rho^2 d\phi^2) + 2\rho\cos(2\phi)\, d\rho\, d\phi),\end{aligned} \tag{25}$$

where the following trigonometric relations were used $(\cos^2(\phi) - \sin^2(\phi)) = \cos(2\phi),\ 2\sin(\phi)\cos(\phi) = \sin(2\phi)$. We have also transformed from the cartesian unit polarization tensors $\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}},\ \mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{y}}}}$ to the corresponding cylindrical ones $\mathbf{e}_{+_{\hat{\rho}\hat{\rho}}},\ \mathbf{e}_{+_{\hat{\rho}\hat{\phi}}}$ by using:

(1) The unit polarization tensors in the $(\mathbf{e}_{\hat{\mathbf{x}}},\ \mathbf{e}_{\hat{\mathbf{y}}},\ \mathbf{e}_{\hat{\mathbf{z}}})$ system $\mathbf{e}_{+_{\hat{\mathbf{x}}\hat{\mathbf{x}}}} = \mathbf{e}_{\hat{\mathbf{x}}}\otimes\mathbf{e}_{\hat{\mathbf{x}}} - \mathbf{e}_{\hat{\mathbf{y}}}\otimes\mathbf{e}_{\hat{\mathbf{y}}} = -\mathbf{e}_{+_{\hat{\mathbf{y}}\hat{\mathbf{y}}}},\ \ \mathbf{e}_{\times_{\hat{\mathbf{x}}\hat{\mathbf{y}}}} = \mathbf{e}_{\hat{\mathbf{x}}}\otimes\mathbf{e}_{\hat{\mathbf{y}}} + \mathbf{e}_{\hat{\mathbf{y}}}\otimes\mathbf{e}_{\hat{\mathbf{x}}} = \mathbf{e}_{\times_{\hat{\mathbf{y}}\hat{\mathbf{x}}}}$ (see Eqs. (4)).

(2) The transformation relations from the $(\mathbf{e}_{\hat{x}}, \mathbf{e}_{\hat{y}}, \mathbf{e}_{\hat{z}})$ coordinate system to the cylindrical one $(\mathbf{e}_{\hat{\rho}}, \mathbf{e}_{\hat{\phi}}, \mathbf{e}_{\hat{z}})$ (Spiegel, 1959)

$$\mathbf{e}_{\hat{\mathbf{x}}} = \cos(\phi)\mathbf{e}_{\hat{\rho}} - \sin(\phi)\mathbf{e}_{\hat{\phi}},\quad \mathbf{e}_{\hat{\mathbf{y}}} = \sin(\phi)\mathbf{e}_{\hat{\rho}} + \cos(\phi)\mathbf{e}_{\hat{\phi}},\quad \mathbf{e}_{\hat{\mathbf{z}}} = \mathbf{e}_{\hat{\mathbf{z}}},$$

and (3) the triginometric identities $(\cos^2(\phi) - \sin^2(\phi)) = \cos(2\phi)$, $2\cos(\phi)\sin(\phi) = \sin(2\phi)$. Thus, the cylindrical unit polarization tensors $\mathbf{e}_{+_{\hat{\rho}\hat{\rho}}}, \mathbf{e}_{+_{\hat{\rho}\hat{\phi}}}$ in Eq. (25) are, respectively, given by

$$\begin{aligned}\mathbf{e}_{+_{\hat{\rho}\hat{\rho}}} &= \cos(2\phi)(\mathbf{e}_{\hat{\rho}}\otimes\mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}}\otimes\mathbf{e}_{\hat{\phi}}) - \sin(2\phi)(\mathbf{e}_{\hat{\rho}}\otimes\mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}}\otimes\mathbf{e}_{\hat{\rho}}) \\ \mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}} &= \sin(2\phi)(\mathbf{e}_{\hat{\rho}}\otimes\mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}}\otimes\mathbf{e}_{\hat{\phi}}) + \cos(2\phi)(\mathbf{e}_{\hat{\rho}}\otimes\mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}}\otimes\mathbf{e}_{\hat{\rho}})\end{aligned} \tag{26}$$

We assume here, as in Brill and Lindquist (1963); Eppley (1977), a no-rotation case so that $h^{TT}_{\hat{\rho}\hat{\phi}}$ is identically zero. Thus, removing from Eq. (25) the $d\rho d\phi$ part one obtains

$$\begin{aligned}(ds^{TT})^2_{(\hat{\rho},\hat{\phi},\hat{\mathbf{z}})} &= h^{TT}_{\hat{\rho}\hat{\rho}}d^2\rho + h^{TT}_{\hat{\phi}\hat{\phi}}d^2\phi \\ &= \cos(kz - ft)(A_+\mathbf{e}_{+_{\hat{\rho}\hat{\rho}}}\cos(2\phi) + A_\times\mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}}\sin(2\phi))((d\rho^2 - \rho^2 d\phi^2))\end{aligned} \tag{27}$$

We, now, find the embedding of the holographic image and begin by assuming, as for the trapped surfaces discussed in Eppley (1977), that its metric in the small surface A (see Fig. 2) is that of a surface of rotation $z(x, y)$ related to Euclidean space. That is, one may write

$$x = a(\rho)\cos(\phi),\quad y = a(\rho)\sin(\phi),\quad z = b(\rho) \tag{28}$$

Thus, using the expressions for $h^{TT}_{\hat{\rho}\hat{\rho}}$ and $h^{TT}_{\hat{\phi}\hat{\phi}}$ from Eq. (27) one may write the metric of the holographic image as

$$\begin{aligned}
ds^2 &= dx^2 + dy^2 + dz^2 = (a^2(\rho)_\rho + b^2(\rho)_\rho)\, d\rho^2 + a^2(\rho)\, d\phi^2 \\
&= h^{TT}_{\hat{\rho}\hat{\rho}} d^2\rho + h^{TT}_{\hat{\phi}\hat{\phi}} d^2\phi = \cos(kz - ft)(A_+ \mathbf{e}_{+_{\hat{\rho}\hat{\rho}}} \cos(2\phi) + A_\times \mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}} \sin(2\phi)) \\
&\quad \times (d\rho^2 - \rho^2 d\phi^2) = \cos(kz - ft)\left[\frac{\sin(4\phi)}{2}(A_\times - A_+)\left(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\right)\right. \\
&\quad \left. + (A_+ \cos^2(2\phi) + A_\times \sin^2(2\phi))(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}})\right]\left(d^2\rho - \rho^2 d\phi^2\right) \qquad (29)
\end{aligned}$$

where $a_\rho(\rho),\ b_\rho(\rho)$ denote the first derivatives of a, b with respect to ρ and the last result is obtained by substituting from Eq. (26) for $\mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}},\ \mathbf{e}_{+_{\hat{\rho}\hat{\rho}}}$. Note that when the amplitudes $A_\times$, A_+ are equal the expression $(A_+ \mathbf{e}_{+_{\hat{\rho}\hat{\rho}}} \cos(2\phi) + A_\times \mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}} \sin(2\phi))$ is considerably simplified and becomes

$$(A_+ \mathbf{e}_{+_{\hat{\rho}\hat{\rho}}} \cos(2\phi) + A_\times \mathbf{e}_{\times_{\hat{\rho}\hat{\phi}}} \sin(2\phi)) = A_+(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}})$$

The quantities $a(\rho)$, $a_\rho(\rho)$ and $b(\rho)$ which defines the intrinsic geometry of the holographic image upon the small area A are determined from Eq. (29) as

$$\begin{aligned}
a(\rho) &= \rho\left[\cos(kz - ft)\left\{\frac{\sin(4\phi)}{2}(A_+ - A_\times)\left(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\right)\right.\right. \\
&\quad \left.\left. + \left(A_+ \cos^2(2\phi) + A_\times \sin^2(2\phi)\right)(\mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}} - \mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}})\right\}\right]^{\frac{1}{2}} \\
a(\rho)_\rho &= \left[\cos(kz - ft)\left\{\frac{\sin(4\phi)}{2}(A_+ - A_\times)\left(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\right)\right.\right. \\
&\quad \left.\left. + \left(A_+ \cos^2(2\phi) + A_\times \sin^2(2\phi)\right)(\mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}} - \mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}})\right\}\right]^{\frac{1}{2}} \qquad (30) \\
b(\rho) &= \int d\rho\left[\cos(kz - ft)\left\{\frac{\sin(4\phi)}{2}(A_\times - A_+)\left(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\phi}} + \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\rho}}\right)\right.\right. \\
&\quad \left. + \left(A_+ \cos^2(2\phi) + A_\times \sin^2(2\phi)\right)(\mathbf{e}_{\hat{\rho}} \otimes \mathbf{e}_{\hat{\rho}} - \mathbf{e}_{\hat{\phi}} \otimes \mathbf{e}_{\hat{\phi}})\} - a^2(\rho)_\rho\right]^{\frac{1}{2}}
\end{aligned}$$

Note that the geometry of the small surface A determined from the former quantities $a(\rho)$, $a(\rho)_\rho$, $b(\rho)$ is that of the subject GW $h^{TT}_{S_{\hat{\rho},\hat{\phi},\hat{z}}}$.

5. CONCLUDING REMARKS

We have continued and expanded our former discussion (Bar, 2005) regarding the holographic properties of the GW's which, theoretically, results from a comparison between optics holography and the linearized version of general relativity. It is argued that the outcome of the gravitational holographic process, which is the additional curving of the relevant spacetime region compared to neighbouring regions may be considered as if the spacetime geometry of the passing GW was implanted upon this spacetime region. Thus, this finite spacetime region which carry the geometry of the passing GW may be thought of as a surface formed by this GW which corresponds to the trapped surfaces (Eppley, 1977; Brill, 1959, 1964; Brill and Hartle, 1964; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981) which are also; (1) imprinted upon spacetime by passing GW's and (2) carry the same spacetime geometry as that of the generating GW's. Moreover, this correspondence is more emphasized by noting that it has already been found (Tipler, 1980; Urtsever, 1988a,b) that the collision between two plane GW's results in the overall strenghtening of them (Tipler, 1980; Urtsever, 1988a,b) (corresponds to constructive interference) and the formation of a singularity, which is generally surrounded by a surface, in the involved region.

Now, although these trapped surfaces are generally discussed in the literature (Eppley, 1977; Alcubierre *et al.*, 2000; Gentle *et al.*, 1998; Gentle, 1999; Miyama, 1981; Beig and Murchadha, 1991; Abrahams and Evans, 1992) as formed from strong GW's which do not conform to the linearized version of general relativity and the resulting weak GW's discussed here it should be noted, as mentioned, that we only discuss here the situation *in the presence* of these GW's. That is, as already emphasized in Alcubierre *et al.* (2000); Gentle *et al.* (1998); Gentle (1999); Miyama (1981), weak GW's do not leave any impression upon the relevant spacetime region after passing and disappearing from it but they obviously influence this finite region during their presence in it. Thus, although the GW's, including the weak ones, move with the velocity c of light one may discuss either the situation at the very instant at which this wave passes this region or the situation at which a large number of similar waves are passing one after the other through this region. Moreover, we discuss here the constructive interference of two GW's which have a larger influence upon spacetime compared to that of the single one. And indeed it is shown here that these GW's does form *during their presence* in the relevant spacetime region a trapped surface which, as mentioned, remains and stays so long as the forming waves does not disappear from it.

We note that unlike the EM holographic images which are 3-D surfaces seen by the naked eye and so may be directly discussed in observational terms such as length, distance, intensity etc the gravitational holographic images, actually, denote changes of spacetime curvature and topology (Finkelstein and Rodriguez, 1984; Sorkin, 1986) which can not be directly observed and measured (see discussion in Bar, 2005). Moreover, the efforts to experimentally detect (Abbott *et al.*, 2004; Acernese *et al.*, 2002; Danzmann *et al.*, 1995; Ando and the TAMA collaboration, 2002) GW's did not succeed thus far. However, this does not prevent us to use the equivalence principle (Misner *et al.*, 1973; Bergmann, 1976) and describe any physical event in terms of either curved spacetime in vacuum without resorting to any physical interactions or in terms of physical particles subjected to physical correlations and forces in otherwise flat spacetime.

In order to illustrate our meaning we refer to the linearized GW discussed here and in Bar (2005) which changes spacetime curvature in the region passed by it so that, as shown in Fig. 1, n test particles (TP) arrayed in circular form is transformed to an elliptic one. This process may be discussed from the point of view of curved spacetime so that the n particles are represented by n Einstein-Rosen bridges (Einstein and Rosen, 1935) each of them is surrounded by an intrinsic trapped (minimal) surface (Eppley, 1977; Brill, 1959, 1964; Brill and Hartle, 1964). And their being now in locations in which they are generally closer to each other (as in an ellipse array) is described by an additional trapped surface (Brill, 1959, 1964; Brill and Hartle, 1964) (the $n+1$-st one) which connect all the n Einstein-Rosen bridges. One may alternatively describe this process by saying that n physical particles, which are always situated in flat spacetime, have undergone some **physical correlation** which changes their initial spatial array from a circular form to an elliptic one.

APPENDIX A: THE LINEARIZED GRAVITATIONAL WAVE

In this appendix we refer to a TP which fall freely along the geodesic $\mathcal{A}$ and watches another TP falling freely along a neighbouring geodesic $\mathcal{B}$. Referring to the general case of a coordinate system in which the basis e_β changes arbitrarily but smoothly from point to point and denoting the tangent vector to the geodesic $G(\hat{n}, \tau)$ as $\mathbf{u} = \frac{\partial G(\hat{n},\tau)}{\partial \tau}$ one may write the velocity of the TP along $\mathcal{B}$ relative to that along $\mathcal{A}$ as

$$\nabla_{\mathbf{u}}\mathbf{n} = (n^\beta{}_{;\gamma}\, u^\gamma)e_\beta \tag{A.1}$$

$n^\beta{}_{;\gamma}$ is the covariant derivative of n^β given by Misner *et al.* (1973); Bergmann (1976)

$$n^\beta{}_{;\gamma} = \frac{dn^\beta}{dx^\gamma} + \Gamma^\beta_{\mu\gamma} n^\mu, \tag{A.2}$$

where

$$\Gamma^{\beta}_{\mu\gamma} = g^{\nu\beta}\Gamma_{\nu\mu\gamma} = \frac{1}{2}g^{\nu\beta}(g_{\nu\mu,\gamma} + g_{\nu\gamma,\nu} - g_{\mu\gamma,\nu}) \tag{A.3}$$

The expression between the circular parentheses in (A.1) represents the components of $\nabla_{\mathbf{u}}\mathbf{n}$ and is denoted by $\frac{Dn^{\beta}}{d\tau}$. Thus, using Eq. (A.1) one may write (Misner *et al.*, 1973)

$$\frac{Dn^{\beta}}{d\tau} = n^{\beta};_{\gamma}\, u^{\gamma} = \frac{dn^{\beta}}{d\gamma} + \Gamma^{\beta}_{\mu\gamma}n^{\mu}\frac{dx^{\gamma}}{d\tau} \tag{A.4}$$

The acceleration of the TP along $\mathcal{B}$ relative to that along $\mathcal{A}$ is (Misner *et al.*, 1973)

$$\nabla_{\mathbf{u}}(\nabla_{\mathbf{u}}\mathbf{n}) = -R, \tag{A.5}$$

where R is the Riemann curvature tensor given in component form as (Misner *et al.*, 1973; Bergmann, 1976)

$$R^{\alpha}_{\beta\gamma\delta} = \frac{\partial\Gamma^{\alpha}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial\Gamma^{\alpha}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}_{\mu\gamma}\Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta}\Gamma^{\mu}_{\beta\gamma} \tag{A.6}$$

Equation (A.5) may be written in component form as (Misner *et al.*, 1973)

$$\frac{D^2n^{\alpha}}{d\tau^2} = -R^{\alpha}_{\beta\gamma\delta}u^{\beta}u^{\delta}n^{\gamma} \tag{A.7}$$

Now, returning to the local Lorentz frame represented by the metric (4) one may realize that since, as mentioned, $x^0 = \tau$ on the world line $x^j = 0$ of $\mathcal{A}$ the relation (A.7) reduces to the much simplified form

$$\frac{D^2n^{j}}{d\tau^2} = -R^{j}_{0k0}n^{k} = -R_{j0k0}n^{k} \tag{A.8}$$

We note that the transverse trace-free (TT) coordinate system may move, to first order in the metric perturbation h^{TT}_{jk}, with TP $\mathcal{A}$ and with its proper reference frame. Thus, to first order in h^{TT}_{jk}, one may identify the time t in the coordinate system TT with the proper time τ of the TP $\mathcal{A}$ so as to have $R^{TT}_{j0k0} = R_{j0k0}$. In the last equality the R_{j0k0} at the right is calculated in the proper reference frame of $\mathcal{A}$ and that at the left is calculated in the TT coordinate system where it has been shown (see Eq. (35.10) in Misner *et al.* (1973)) to assume the very simple form of

$$R_{j0k0} = -\frac{1}{2}h^{TT}_{jk,00} \tag{A.9}$$

One may notice that since the TT coordinate system moves with the proper reference frame of TP $\mathcal{A}$ they are both denoted by the same indices $(0, k, j)$ without having to use primed and unprimed indices. Note that since the origin is attached to $\mathcal{A}$'s geodesic the components of the separating vector $\mathbf{n}$ are, actually, the coordinates of $\mathcal{B}$. That is, denoting the coordinates of $\mathcal{A}$ and $\mathcal{B}$ by $x^j_{\mathcal{A}}$ and $x^j_{\mathcal{B}}$ respectively one may write $n^j = x^j_{\mathcal{B}} - x^j_{\mathcal{A}} = x^j_{\mathcal{B}} - 0 = x^j_{\mathcal{B}}$. One may also notice that at $x^j = 0$ the connection coefficients $\Gamma^{\mu}_{\alpha\beta}$ satisfy $\Gamma^{\mu}_{\alpha\beta} = 0$ for all x^0. This entails also the vanishing of $\frac{d\Gamma^{\mu}_{\alpha\beta}}{d\tau}$ thereby the covariant derivative $\frac{D^2 n^j}{d\tau^2}$ becomes ordinary derivative and, using Eq. (A.9), one may write Eq. (A.8) as

$$\frac{d^2 x^j_{\mathcal{B}}}{d\tau^2} = -R_{j0k0} x^k_{\mathcal{B}} = \frac{1}{2}\left(\frac{\partial^2 h^{TT}_{jk}}{\partial t^2}\right) x^k_{\mathcal{B}} \tag{A.10}$$

We choose the initial condition that the test particles at $\mathcal{A}$ and $\mathcal{B}$ were at rest before the wave arrives, that is, $x^j_{\mathcal{B}} = x^j_{\mathcal{B}(0)}$ when $h^{TT}_{jk} = 0$. In this case the solution of Eq. (A.10) is

$$x^j_{\mathcal{B}}(\tau) = x^k_{\mathcal{B}(0)}\left(\delta_{jk} + \frac{1}{2} h^{TT}_{jk}\right)_{at\mathcal{A}}, \tag{A.11}$$

which represents the change in $\mathcal{B}$'s place due to the passing wave as calculated in the proper reference frame of $\mathcal{A}$. The h^{TT}_{jk} at the right hand side of the last equation represents the perturbation caused by advancing gravitational wave which may be assumed to have any form. One may choose any general waveform which may analytically expressed as an infinite expansion of scalar, vector or tensor spherical harmonics (Thorne, 1980b) or one may prefer an exact plane gravitational wave (see Section 35.9 in Misner *et al.* (1973)).

APPENDIX B: The EM Trapped Surface

As emphasized in Misner *et al.* (1973), (see pp. 961–962 there), the EM plane wave equation *"has exactly the form of the equation for the gravitational plane wave"* and *"in the limit of very small wavelength the two solutions are completely indistinguishable. Their metrics are identical" etc*. Thus, in this limit one may use the former process of finding the geometry of the gravitational trapped surface for calculating the corresponding surface related to EM waves. We should only take into account that an EM wave advancing along the z direction is characterized by the unit polarization vectors $\mathbf{e}_{\hat{\mathbf{x}}}$, $\mathbf{e}_{\hat{\mathbf{y}}}$. Thus, as in Eq. (18), one may write the EM metric components $h^{EM}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}$, $h^{EM}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}$ as

$$\begin{aligned} h^{EM}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} &= \Re\left(A_x \mathbf{e}_{\hat{\mathbf{x}}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = A_x \mathbf{e}_{\hat{\mathbf{x}}} \cdot \cos(kz - ft) \\ h^{EM}_{\hat{\mathbf{y}}\hat{\mathbf{y}}} &= \Re\left(A_y \mathbf{e}_{\hat{\mathbf{y}}} e^{-ift} e^{ik\mathbf{r}\hat{\mathbf{z}}}\right) = A_y \mathbf{e}_{\hat{\mathbf{y}}} \cdot \cos(kz - ft), \end{aligned} \tag{B.1}$$

where A_x, A_y are the x and y amplitudes of the EM wave which are considered to be equal as the corresponding xx and yy components of the amplitude A_+ (see pp. 952–953 in Misner *et al.* (1973)). Using the last equation one may write the metric in the $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ coordinate system as

$$(ds^{EM})^2_{(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})} = h^{EM}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}dx^2 + h^{EM}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}dy^2 = A_x \cos(kz - ft)(\mathbf{e}_{\hat{\mathbf{x}}}dx^2 + \mathbf{e}_{\hat{\mathbf{y}}}dy^2) \quad \text{(B.2)}$$

Using the transformation relations $x = \rho\cos(\phi)$, $y = \rho\sin(\phi)$, $z = z$, $\mathbf{e}_{\hat{\mathbf{x}}} = \cos(\phi)\mathbf{e}_{\hat{\rho}} - \sin(\phi)\mathbf{e}_{\hat{\phi}}$, $\mathbf{e}_{\hat{\mathbf{y}}} = \sin(\phi)\mathbf{e}_{\hat{\rho}} + \cos(\phi)\mathbf{e}_{\hat{\phi}}$, $\mathbf{e}_{\hat{\mathbf{z}}} = \mathbf{e}_{\hat{\mathbf{z}}}$, we first express the metric from Eq. (B.2) in the cylindrical coordinate system $(\hat{\rho}, \hat{\phi}, \hat{\mathbf{z}})$ and use, as for the gravitational case (see the discussion after Eq. (21)), the no-rotation assumption for which $h^{EM}_{\hat{\rho}\hat{\phi}}d\rho d\phi$ is identically zero so that

$$\begin{aligned}(ds^{EM})^2_{(\hat{\rho},\hat{\phi},\hat{\mathbf{z}})} = h^{EM}_{\hat{\rho}\hat{\rho}}d\rho^2 + h^{EM}_{\hat{\phi}\hat{\phi}}d\phi^2 = A_{\rho,\phi}\cos(kz - ft)\\ \times[\{(\cos^3(\phi) + \sin^3(\phi))\mathbf{e}_{\hat{\rho}} + (\sin^2(\phi)\cos(\phi)\\ - \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\phi}}\}d\rho^2 + \{(\sin^2(\phi)\cos(\phi)\\ + \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\rho}} + (\cos^3(\phi) - \sin^3(\phi))\mathbf{e}_{\hat{\phi}}\}\rho^2 d\phi^2]\end{aligned} \quad \text{(B.3)}$$

Using again, as in Eq. (23), the relations $x = a(\rho)\cos(\phi)$, $y = a(\rho)\sin(\phi)$, $z = b(\rho)$ one may write the metric as

$$\begin{aligned}(ds^{EM})^2 &= dx^2 + dy^2 + dz^2 = (a^2(\rho)_\rho + b^2(\rho)_\rho)d\rho^2 + a^2(\rho)d\phi^2\\ &= h^{EM}_{\hat{\rho}\hat{\rho}}d^2\rho + h^{EM}_{\hat{\phi}\hat{\phi}}d^2\phi = \cos(kz - ft)\\ &\times[\{(\cos^3(\phi) + \sin^3(\phi))\mathbf{e}_{\hat{\rho}} + (\sin^2(\phi)\cos(\phi) - \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\phi}}\}\\ &\times d\rho^2 + \{(\sin^2(\phi)\cos(\phi) + \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\rho}} + (\cos^3(\phi)\\ &- \sin^3(\phi))\mathbf{e}_{\hat{\phi}}\}\rho^2 d\phi^2]\end{aligned} \quad \text{(B.4)}$$

The appropriate expressions $a(\rho)$, $a(\rho)_\rho$, $b(\rho)$ which determine the geometry of the relevant trapped surface are obtained from Eq. (B.4) as

$$\begin{aligned}a(\rho) &= \rho[\cos(kz - ft)\{(\sin^2(\phi)\cos(\phi) + \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\rho}}\\ &+ (\cos^3(\phi) - \sin^3(\phi))\mathbf{e}_{\hat{\phi}}\}]^{\frac{1}{2}}\\ a(\rho)_\rho &= [\cos(kz - ft)\{(\sin^2(\phi)\cos(\phi) + \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\rho}}\\ &+ (\cos^3(\phi) - \sin^3(\phi))\mathbf{e}_{\hat{\phi}}\}]^{\frac{1}{2}}\\ b(\rho) &= \int d\rho[\cos(kz - ft)\{(\cos^3(\phi) + \sin^3(\phi))\mathbf{e}_{\hat{\rho}}\\ &+ (\sin^2(\phi)\cos(\phi) - \cos^2(\phi)\sin(\phi))\mathbf{e}_{\hat{\phi}}\} - a^2(\rho)_\rho]^{\frac{1}{2}}\end{aligned} \quad \text{(B.5)}$$

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